# 1 stepped pressure equilibrium code : sw00ad

## Contents



## 1.1 outline

1. Constructs spectrally-condensed Fourier representation of interfaces using a stream function.

### 1.1.1 angle transformation

2. The geometry of each interface is given (on input) as

$$\begin{array}{rcl} R(\theta,\zeta) & = & \sum_i R_i \cos(m_i\theta - n_i\zeta), \\ Z(\theta,\zeta) & = & \sum_i Z_i \sin(m_i\theta - n_i\zeta). \end{array} \tag{1}$$

3. A new angle, $\bar{\theta}$, shall be introduced via a stream function, $\lambda(\bar{\theta},\zeta)$, according to

$$\theta = \bar{\theta} + \sum_j \lambda_j \sin(m_j\bar{\theta} - n_j\zeta), \tag{2}$$

where the $\lambda_j$ are, as yet, unknown degrees of freedom.

4. The Fourier harmonics in the new angle are

$$\bar{R}_k = \oint\!\!\oint d\bar{\theta}d\zeta \; R\cos(m_k\bar{\theta} - n_k\zeta), \tag{3}$$

$$\bar{Z}_k = \oint\!\!\oint d\bar{\theta}d\zeta \; Z\sin(m_k\bar{\theta} - n_k\zeta), \tag{4}$$

where, by combining Eq.(1) and Eq.(2), it is understood that $R \equiv R(\bar{\theta},\zeta)$and $Z \equiv Z(\bar{\theta},\zeta)$.

5. The spectral-width (in the new angle) is defined

$$M = \frac{1}{2}\sum_k (m_k^p + n_k^q)\left(\bar{R}_k^2 + \bar{Z}_k^2\right), \tag{5}$$

where $m_k^p = 0$ for $m_k = 0$, and $n_k^q = 0$ for $n_k = 0$, and where $p \equiv$`pwidth` and $q \equiv$`qwidth` are given on input.

6. The variation in spectral-width due to variations, $\delta\lambda_j$ is

$$\frac{\partial M}{\partial \lambda_j} = \sum_k (m_k^p + n_k^q)\left(\bar{R}_k \frac{\partial \bar{R}_k}{\partial \lambda_j} + \bar{Z}_k \frac{\partial \bar{Z}_k}{\partial \lambda_j}\right). \tag{6}$$

### 1.1.2 numerical implementation

7. This routine seeks a zero of a vector function, $\mathbf{F}(\boldsymbol{\lambda})$, where $F_j \equiv \partial M/\partial\lambda_j$.

8. The NAG routine `c05nbf` is employed (This routine uses function values only: perhaps the derivatives could be calculated and more efficient routines enabled.)

9. It is probably preferable to use `E04LYF`.

10. The condensed representation is only accepted if $|\partial M/\partial\lambda_j| <$`small`.

11. Differentiating the Fourier harmonic $\bar{R}_k$ with respect to $\lambda_j$ is equivalent to Fourier decomposing the derivative:

$$\frac{\partial \bar{R}_k}{\partial \lambda_j} \equiv \left( \frac{\partial \bar{R}}{\partial \lambda_j} \right)_k, \tag{7}$$

$$\frac{\partial \bar{Z}_k}{\partial \lambda_j} \equiv \left( \frac{\partial \bar{Z}}{\partial \lambda_j} \right)_k. \tag{8}$$

### 1.1.3 comments

1. The quantity $(m_k^p + n_k^q)$ is saved in the internal global variable `mpnq`, which is defined in `al00aa`.

sw00ad.h last modified on 2012-12-18 ;